The Long Term Behaviour of Leveraged ETFs

There is a big myth about leveraged ETFs that has been recently propagated in the media. This article corrects the myth and explains the faulty reasoning that gave rise to it.
The myth is:

Leveraged ETFs are not suitable for long term buy and hold

This myth is expressed in various ways. Some quotes from the internet about leveraged ETFs:
"unsuitable for buy-and-hold investing," "leveraged ETFs are bound to deteriorate," "over time the compounding will kill," "leveraged ETFs verge on insanity," "levered ETFs are toxic," "levered ETFs [are] a horrible idea," "…practically guarantees losses," "in the long run [investors] are almost sure to lose money," "anyone holding these funds for the long term is an uneducated lame-brain." "Warning: Leveraged and Inverse ETFs Kill Portfolios."

There is even an article comparing these ETFs to swine flu.

These claims are not backed up with mathematics and data. This article rectifies that deficiency and finds that the claims are false.

The explanation popularly given for this myth is that volatility eats away at long term returns. If this were true then non-leveraged funds would also not be suitable for buy and hold because they too suffer from volatility. This web article is a synopsis of a full academic mathematical working paper that is in progress and will be released on this web site later this year.

What is a leveraged ETF?

A leveraged ETF is defined for our purposes to be any ETF that promises returns of a multiple of some benchmark return on a *daily* basis where that multiple is specified in the prospectus.

For example if an ETF promises a return of 2 times the S&P 500 index then if the S&P 500 index goes up 1.2% in one day the ETF will go up 2 x 1.2% = 2.4%.

The salient point about this definition is that the multiple may be any number such as -3 or 2.5 *and includes the multiple 1*, and that the return is marked to the benchmark *daily*, not annually. This is true even if the return is measured annually. Including the multiple 1 is done for mathematical convenience – most people would not call such an ETF leveraged but technically it is an ETF with leverage 1.

Origin of the Myth

Daily volatility hurts the returns of leveraged ETFs (including those with leverage 1). This is due to the equality

$$(1 - x)(1 + x) = 1 - x^2$$

Suppose the market goes down by x and then the next day it goes up by x. For example if $x = 0.05$ then the market goes up by 5% then down by 5%. Then the net result is that the market has gone to (1-0.05) times (1+0.05) = 0.9975 which is a drop of 0.0025 or 0.25%.

That's not fair! The market has gone down by 5% then up by 5% but our ETF that has a leverage of 1 has gone down by 0.25%. Doggone it!

This drop always occurs because x^2 is always positive and the sign in front is negative. So whenever the market has volatility we lose money. We call this *volatility drag*.

The larger x is the larger x^2 is so the larger the volatility drag. For a leveraged ETF the leverage multiplies x and so multiplies the volatility drag. Even an ETF with a leverage of 1 has volatility drag.

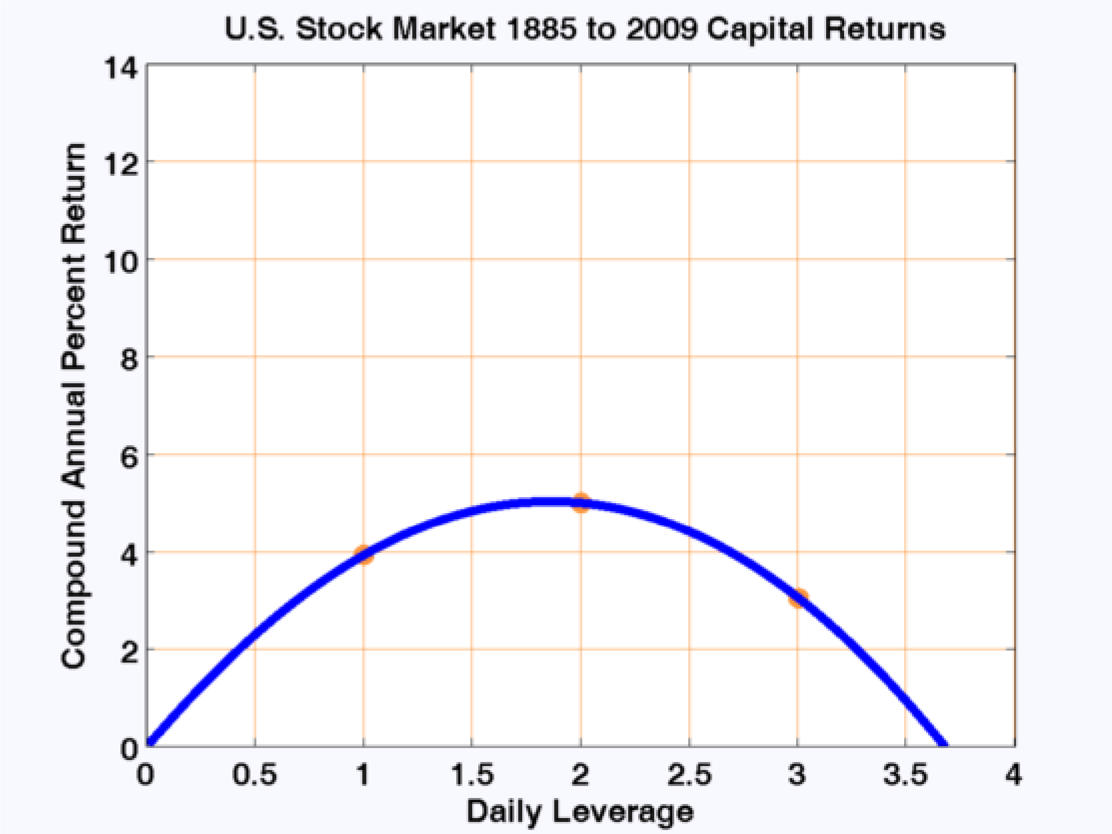
The myth has resulted from the belief that volatility drag will drag any leveraged ETF down to zero given enough time. But we know that leverage of 1 (i.e. no leverage) is safe to hold forever *even though leverage 1 still has volatility drag*. If 1 times leverage is safe then is 1.01 times leverage safe? Is 1.1 times safe? What's so special about 2 times? Where are you going to draw the line between safe and unsafe?

Maybe 2 times is safe. Why shouldn't an ETF with leverage 2 still be suitable for holding forever?

It turns out that there may be a reason but it's not volatility drag.

The Myth is Easily Shown to be False

The chart below shows 135 years worth of daily US index prices going back to 16 February 1885. Yes, that's right, back to 1885. The construction of this index is described in Schwert (1990) and the index used is the capital index (no dividends reinvested). S&P 500 data from Yahoo.com has been used to bring the data up to 2009.



The orange circles show popular leverage rates 1, 2, 3, and just for show, 4. It can be seen that increasing leverage from zero to 1 increases the annualised return as would be expected. But then, contrary to what the myth propagators say, increasing the leverage even further still keeps increasing the returns.

There is nothing magic about the leverage value 1. There is no mathematical reason for returns to suddenly level off at that leverage. The myth propagators are wrong. **Leveraged ETFs can be held long term** (unless you think that 135 years isn't long term).

We can see that returns do drop off once leverage reaches about 2. That is the effect of volatility drag.

What the myth propagators have forgotten is that there are two factors that decide leveraged ETF returns: benchmark returns and benchmark volatility. If the benchmark has a positive return then leveraged exposure to it is good and *compensates for volatility drag*. Since the return is a multiple of leverage and the drag a multiple of the leverage squared then eventually the drag overwhelms the extra return obtained through leverage. So there is a limit to the amount of leverage that can be used.

The Formula for the ETF Long Term Return

The formula for the long term compound annual growth rate of a leveraged ETF cannot be written in terms of just the benchmark return and volatility. It also involves terms containing the skewness and kurtosis of the benchmark. Its derivation and form is published in the paper that accompanies this article. It does not assume that benchmark returns are Gaussian or that returns are continuous as do formulae derived using Ito's lemma.

But it turns out that for the world's stock markets and for low levels of leverage (up to about 3) the formula can be approximated by this formula:

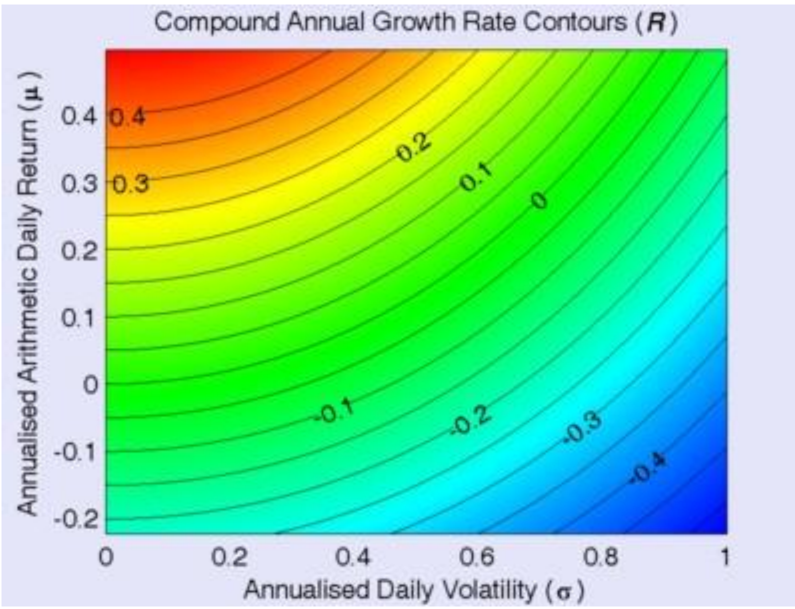
$$R = k\mu - \tfrac{1}{2}k^2\sigma^2/(1 + k\mu)$$

where R is the compound daily growth rate of the ETF, k is the ETF leverage, μ is the mean daily return of the benchmark, and σ is the daily volatility (i.e. standard deviation) of the daily return of the benchmark.

R is the quantity you use to calculate the long term buy-and-hold return of the ETF. You can see from the formula that if the volatility is zero then $R = k\mu$ so that the return of the ETF is k times the return of the benchmark. The $\tfrac{1}{2}k^2\sigma^2/(1 + k\mu)$ term is the volatility drag. Since $k^2\sigma^2$ is always positive and $(1 + k\mu)$ is always close to 1 then the volatility drag is always positive. Unfortunately.

R is a quadratic function of *k* with a negative coefficient for the square term. That means we will always get the parabola shape shown above and we will always have a maximum for some value of *k*. Some algebra shows that the maximum is at $k = \mu/\sigma^2$. This clearly shows the return/volatility trade-off that determines the optimal leverage.

This formula (actually the more accurate version including skewness and kurtosis) is discussed at depth in the full paper. It is a formula that occurs in an appropriate form in the Kelly Criterion and Merton's Portfolio Problem. Its appearance here as the result of an optimisation is no surprise.
The following chart plots *R* versus μ and σ. All three quantities have been annualised since most people are used to thinking in annual returns but they are still daily quantities. The chart can be tricky to understand. But the important point to note is that for a given Daily Return as you move horizontally rightwards on the chart in the direction of increasing Daily Volatility the return *R* becomes more blue – i.e. *R* decreases. This is the effect of volatility drag.



The benefit of this chart is that you can plot leveraged ETFs on the chart simultaneously for all leverages *k*. The numbers 1 to 4 on the chart in the next section show where the points for leverage 1 to 4 lie. Since daily volatility and daily return both increase linearly with *k* then the varying leverages draw out straight lines across the chart. Since the *R* contours are curved the straight lines have to cross the curves. Therefore every line has an optimum value of *R*. That is, there is an optimal leverage for which the long term return is maximised. And that optimum leverage is almost always greater than 1.

Conclusion

Leveraged ETFs can be held long term provided the market has enough return to overcome volatility drag. It usually does. For most markets in recent times the optimal leverage is about 2. But some markets and time frames will reward a leverage of up to 3. No markets will reward a leverage of 4.
Myth busted